

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^2 + b^2 = (a + b)^2 - 2ab = (a - b)^2 + 2ab$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a - b)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$$

$$a + b + c = 0 \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$\text{Last term} = \frac{(\text{middle term})^2}{4 \times \text{First term}}$$

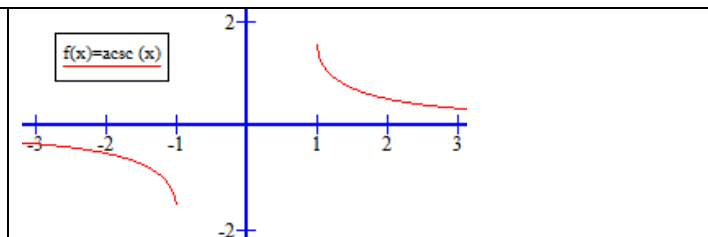
$$\text{Frist term} = \frac{(\text{middle term})^2}{4 \times \text{last term}}$$

$$\text{middle term} = \pm 2\sqrt{(\text{Frist term}) \times (\text{last term})}$$

| FUNCTION | RIGHT ANGLED TRIANGLE | UNIT CIRCLE | RANGE | SET OF ZEROES | PRINCIPAL PERIOD | TYPE OF FUNCTION | GRAPH |
|----------|---|--|--------------|---|------------------|-----------------------|-------|
| cosine | $\frac{\text{Adjacent Side}}{\text{Hypotenuse}}$ | $\cos: \mathbb{R} \rightarrow \mathbb{R},$ $\cos\theta = (\text{gof})(\theta)$ $= g(f(x))$ $= g(x, y)$ $= x$ | $[-1, 1]$ | $\{(2K \pm 1) \frac{\pi}{2} / k \in \mathbb{Z}\}$ | 2π | many-one, on to, even | |
| sine | $\frac{\text{Opposite Side}}{\text{Hypotenuse}}$ | $\sin: \mathbb{R} \rightarrow \mathbb{R},$ $\sin\theta = (\text{gof})(\theta)$ $= g(f(x))$ $= g(x, y)$ $= y$ | $[-1, 1]$ | $\{k\pi / k \in \mathbb{Z}\}$ | 2π | many-one, on to, odd | |
| tan | $\frac{\text{Opposite Side}}{\text{Adjacent Side}}$ | $\tan: \mathbb{R} - \{(2K \pm 1) \frac{\pi}{2} / k \in \mathbb{Z}\} \rightarrow \mathbb{R},$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ | \mathbb{R} | $\{k\pi / k \in \mathbb{Z}\}$ | π | many-one, on to, odd | |

| FUNCTION | RIGHT ANGLED TRIANGLE | UNIT CIRCLE | RANGE | SET OF ZEROES | PRINCIPAL PERIOD | TYPE OF FUNCTION | GRAPH |
|---------------------------------|---|---|------------------------|--|------------------|-----------------------|-------|
| cot | $\frac{\text{Adjacent Side}}{\text{Opposite Side}}$ | tan: $\mathbb{R} - \{k\pi/k \in \mathbb{Z}\}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$ | \mathbb{R} | $\{(2K \pm 1)\frac{\pi}{2}/k \in \mathbb{Z}\}$ | π | many-one, on to, odd | |
| $\sec = \frac{1}{\cos}$ | $\frac{\text{Hypotenuse}}{\text{Adjacent Side}}$ | tan: $\mathbb{R} - \{(2K \pm 1)\frac{\pi}{2}/k \in \mathbb{Z}\} \rightarrow \mathbb{R}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ | $\mathbb{R} - (-1, 1)$ | \emptyset | 2π | many-one, on to, even | |
| $\text{cosec} = \frac{1}{\sin}$ | $\frac{\text{Hypotenuse}}{\text{Opposite Side}}$ | tan: $\mathbb{R} - \{k\pi/k \in \mathbb{Z}\}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$ | $\mathbb{R} - (-1, 1)$ | \emptyset | 2π | many-one, on to, odd | |

| INVERSE FUNCTION | DOMAIN | RANGE | TYPE OF FUNCTION | GRAPH |
|--|------------------------|-----------------------------------|------------------|-------|
| $\cos^{-1} = \{(y, x)/y = \cos x, x \in [0, \pi], y \in [-1, 1]\}$ | $[-1, 1]$ | $[0, \pi]$ | one-one, onto | |
| $\sin^{-1} = \{(y, x)/y = \sin x, x \in [-\frac{\pi}{2}, \frac{\pi}{2}], y \in [-1, 1]\}$ | $[-1, 1]$ | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ | one-one, onto | |
| $\tan^{-1} = \{(y, x)/y = \tan x, x \in (-\frac{\pi}{2}, \frac{\pi}{2}), y \in \mathbb{R}\}$ | \mathbb{R} | $(-\frac{\pi}{2}, \frac{\pi}{2})$ | one-one, onto | |
| $\cot^{-1} = \{(y, x)/y = \cot x, x \in (0, \pi), y \in \mathbb{R}\}$ | \mathbb{R} | $(0, \pi)$ | one-one, onto | |
| $\sec^{-1} = \{(y, x)/y = \sec x, x \in [0, \pi] - (\frac{\pi}{2}), y \in (-1, 1)\}$ | $\mathbb{R} - (-1, 1)$ | $[0, \pi] - (\frac{\pi}{2})$ | one-one, onto | |

| | | | | |
|---|------------------------|--|-------------------|--|
| $\operatorname{cosec}^{-1} = \{(y, x) /$ $y = \operatorname{cosec} x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] -$ $\{0\}, y \in \mathbb{R} - (-1, 1)\}$ | $\mathbb{R} - (-1, 1)$ | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ | one-one, on to |  |
|---|------------------------|--|-------------------|--|

1. $\sin^{-1} x \neq (\sin x)^{-1} = \frac{1}{\sin x}$
2. $\forall \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \sin^{-1}(\sin \theta) = \theta$ and $\forall x \in [-1, 1], \sin^{-1}(\sin x) = x$.
same as cos, tan, cot, sec, cosec.
3. codomain of sin, cos, tan, cot, sec & cosec function is R.
4. Range of \sin^2 and \cos^2 function are $[0, 1]$.
5. Range of \tan^2 and \cot^2 function are $[0, \infty)$.
6. Range of cosec^2 and \sec^2 function are $[1, \infty)$.
7. All S T C.
8. Increasing function is \uparrow and decreasing function is \downarrow .

| | First Quadrant | Second Quadrant | Third Quadrant | Fourth Quadrant |
|-------|----------------|-----------------|----------------|-----------------|
| sin | \uparrow | \downarrow | \downarrow | \uparrow |
| cos | \downarrow | \downarrow | \uparrow | \uparrow |
| cosec | \downarrow | \uparrow | \uparrow | \downarrow |
| sec | \uparrow | \uparrow | \downarrow | \downarrow |

We assume that tan is \uparrow and cot is \downarrow in all quadrant.

| | | | | | | | | | | | | | |
|---------|----|-------------------------------|----------------------------------|-------------------------------|----------------------|---------------------------------|----------------------|----------------------------------|----------------------|-------------------------------|---------------------------------|-------------------------------|-----------------|
| Degrees | 0° | 15° | 18° | 22.5° | 30° | 36° | 45° | 54° | 60° | 67.5° | 72° | 75° | 90° |
| Radian | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{10}$ | $\frac{\pi}{8}$ | $\frac{\pi}{6}$ | $\frac{\pi}{5}$ | $\frac{\pi}{4}$ | $\frac{3\pi}{10}$ | $\frac{\pi}{3}$ | $\frac{3\pi}{8}$ | $\frac{2\pi}{5}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ |
| sin | 0 | $\frac{\sqrt{6}-\sqrt{2}}{4}$ | $\frac{\sqrt{5}-1}{4}$ | $\frac{\sqrt{2-\sqrt{2}}}{2}$ | $\frac{1}{2}$ | $\frac{\sqrt{10-2\sqrt{5}}}{4}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{6}-\sqrt{2}}{4}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2+\sqrt{2}}}{2}$ | $\frac{\sqrt{10+2\sqrt{5}}}{4}$ | $\frac{\sqrt{6}+\sqrt{2}}{4}$ | 1 |
| cos | 1 | $\frac{\sqrt{6}+\sqrt{2}}{4}$ | $\frac{\sqrt{10+2\sqrt{5}}}{4}$ | $\frac{\sqrt{2+\sqrt{2}}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{5}+1}{4}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{6}-\sqrt{2}}{4}$ | $\frac{1}{2}$ | $\frac{\sqrt{2-\sqrt{2}}}{2}$ | $\frac{\sqrt{5}-1}{4}$ | $\frac{\sqrt{6}-\sqrt{2}}{4}$ | 0 |
| tan | 0 | $2-\sqrt{3}$ | $\frac{\sqrt{25-10\sqrt{5}}}{4}$ | $\sqrt{2}-1$ | $\frac{1}{\sqrt{3}}$ | $\sqrt{5-2\sqrt{5}}$ | 1 | $\frac{\sqrt{25-10\sqrt{5}}}{4}$ | $\sqrt{3}$ | $\sqrt{2}+1$ | $\sqrt{5+2\sqrt{5}}$ | $2+\sqrt{3}$ | ∞ |

| | | | |
|---|---|---|---|
| $\cos^2\theta + \sin^2\theta = 1$ | $\cos^2\theta = 1 - \sin^2\theta$ | $\sin^2\theta = 1 - \cos^2\theta$ | $\theta \in \mathbb{R}$ |
| $\sec^2\theta - \tan^2\theta = 1$ | $1 + \tan^2\theta = \sec^2\theta$ | $\sec^2\theta - 1 = \tan^2\theta$ | $\theta \neq (2k+1)\frac{\pi}{2}, k \in \mathbb{Z}$ |
| $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ | $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ | $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$ | $\theta \neq k\pi, k \in \mathbb{Z}$ |

Addition Formula:- $\alpha, \beta \in \mathbb{R}$

$$\Rightarrow \cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

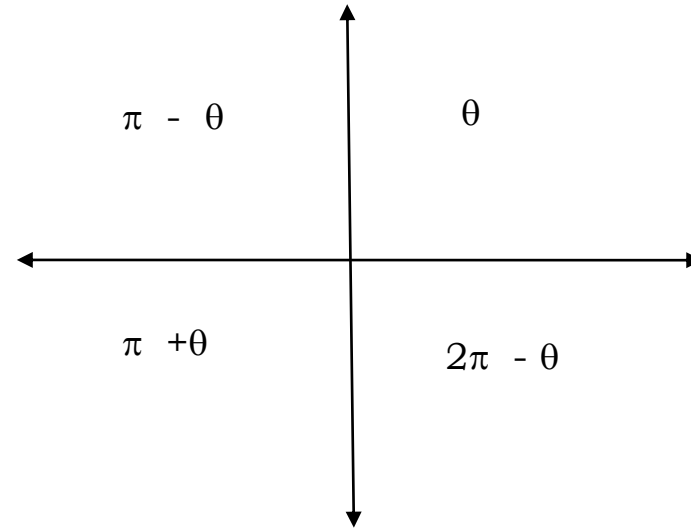
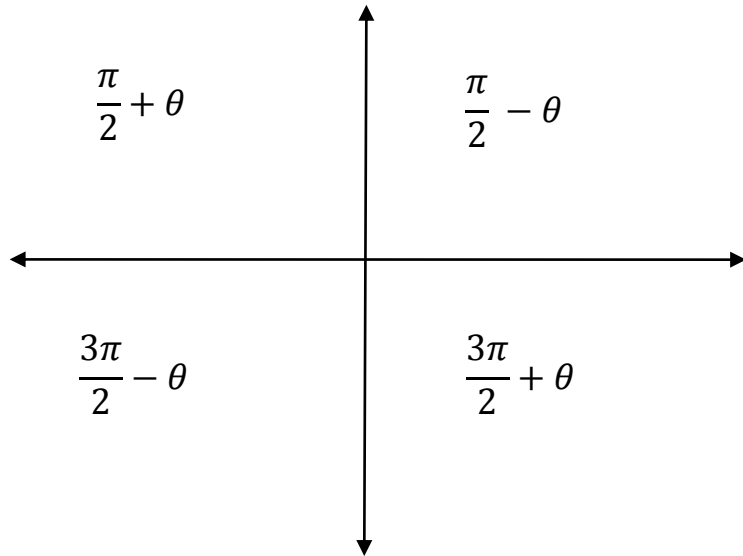
$$\Rightarrow \sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

If $k\pi \pm \theta$, $k \in \mathbb{Z}$ is given then the function remains same.

| | |
|-----------------------------|---|
| $\sin \leftrightarrow \sin$ | $\cos \leftrightarrow \cos$ |
| $\tan \leftrightarrow \tan$ | $\cot \leftrightarrow \cot$ |
| $\sec \leftrightarrow \sec$ | $\operatorname{cosec} \leftrightarrow \operatorname{cosec}$ |

If $\frac{k\pi}{2} \pm \theta$, $k \in \mathbb{Z}$, k is odd integer is given then the function remains change.

| |
|---|
| $\sin \leftrightarrow \cos$ |
| $\tan \leftrightarrow \cot$ |
| $\sec \leftrightarrow \operatorname{cosec}$ |



$$\sin(\alpha + \beta) \cdot \sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\alpha - \cos^2\beta$$

$$\cos(\alpha + \beta) \cdot \cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

The range of $f(\alpha) = a \cos\alpha + b \sin\alpha$, $\alpha \in \mathbb{R}$; $a, b \in \mathbb{R}$, $a^2 + b^2 \neq 0$ is $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$

OR $-r \leq r \cos(\theta - \alpha) \leq r$, \therefore The range is $[-r, r]$.

$$2\sin\alpha \cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta) \rightarrow 1$$

$$2\cos\alpha \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta) \rightarrow 2$$

$$2\cos\alpha \cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta) \rightarrow 3$$

$$2\sin\alpha \sin\beta = -\cos(\alpha + \beta) + \cos(\alpha - \beta) \rightarrow 4$$

Expression of sum or difference as product:

$$\text{let in 1 to 4 } \alpha + \beta = C \text{ and } \alpha - \beta = D, \therefore \alpha = \frac{C+D}{2} \text{ and } \beta = \frac{C-D}{2}$$

$$\sin C + \sin D = 2\sin\frac{C+D}{2} \cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2} \sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2} \cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2} \sin\frac{C-D}{2}$$

$$\sin\alpha = 2\sin\frac{\alpha}{2} \cos\frac{\alpha}{2} = \sqrt{\frac{1 - \cos 2\alpha}{2}} = \frac{2\tan\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}} = \frac{2t}{1 + t^2} \quad \left(\text{take } \tan\frac{\alpha}{2} = t\right)$$

$$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = 2\cos^2\frac{\alpha}{2} - 1 = 1 - 2\sin^2\frac{\alpha}{2} = \sqrt{\frac{1 + \cos 2\alpha}{2}} = \frac{1 - \tan^2\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}} = \frac{1 - t^2}{1 + t^2}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1 - \tan^2\frac{\alpha}{2}}$$

$$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos\alpha}{2}$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos\alpha}{2}$$

$$\tan^2 \frac{\alpha}{2} = \frac{1 - \cos\alpha}{1 + \cos\alpha}$$

INVERSE TRIGONOMETRIC FORMULA : -

$$\sin^{-1}(-x) = -\sin^{-1} x, \quad |x| \leq 1$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x, \quad |x| \leq 1$$

$$\tan^{-1}(-x) = -\tan^{-1} x, \quad x \in \mathbb{R}$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x, \quad x \in \mathbb{R}$$

$$\operatorname{cosec}^{-1}(-x) = \sin^{-1} \frac{1}{x}, \quad |x| \geq 1$$

$$\sec^{-1}(-x) = \cos^{-1} \frac{1}{x}, \quad |x| \geq 1$$

$$\cot^{-1} x = \tan^{-1} \frac{1}{x}, \quad x \geq 1$$

$$\cot^{-1} x = \tan^{-1} \frac{1}{x} + \pi, x < 1$$

Value of inverse functions for complementary numbers :→

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \quad |x| \leq 1$$

$$\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}, \quad |x| \geq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in \mathbb{R}$$

Values for Addition and Subtraction :→

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}, xy < 1$$

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}, xy > 1$$

$$\tan^{-1} x + \tan^{-1} y = \frac{\pi}{2}, xy = 1$$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

Inter-relations of inverse trigonometric function :→

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{x}{\sqrt{1-x^2}}, 0 < x < 1$$

$$\cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{\sqrt{1-x^2}}{x}, 0 < x < 1$$

$$\tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1+x^2}} = \sin^{-1} \frac{x}{\sqrt{1+x^2}}, x > 0$$

Trigonometric Equations :→

The general solution of $\cos x = a$, $|a| \leq 1 \Rightarrow x = 2k\pi \pm \cos^{-1} a, k \in \mathbb{Z}$

$$\therefore \cos x = \cos \theta \Rightarrow x = 2k\pi \pm \theta, k \in \mathbb{Z}$$

The general solution of $\sin x = a$, $|a| \leq 1 \Rightarrow x = k\pi + (-1)^k \sin^{-1} a, k \in \mathbb{Z}$

$$\therefore \sin x = \sin \theta \Rightarrow x = k\pi + (-1)^k \theta, k \in \mathbb{Z}$$

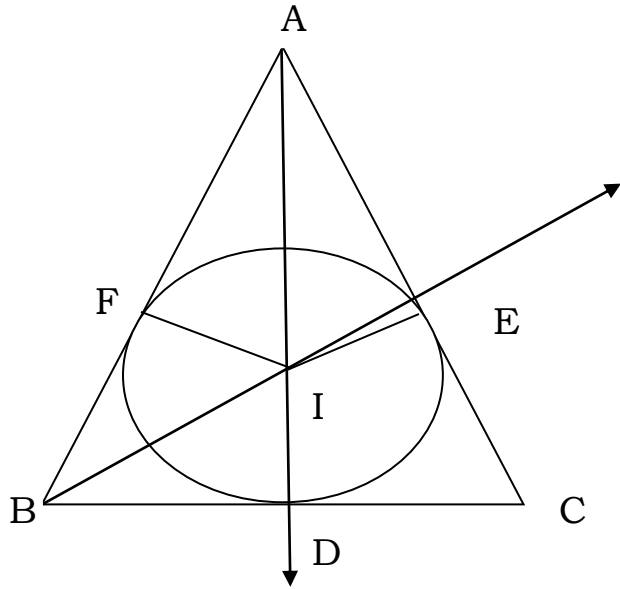
The general solution of $\tan x = a$, $a \in \mathbb{R} \Rightarrow x = k\pi + \tan^{-1} a, k \in \mathbb{Z}$

$$\therefore \tan x = \tan \theta \Rightarrow x = k\pi + \theta, k \in \mathbb{Z}$$

The solution set of $a \cos x + b \sin x = c$, $a^2 + b^2 \neq 0$

$$\Leftrightarrow x = 2k\pi + \alpha \pm \cos^{-1} \frac{c}{r}, k \in \mathbb{Z}, \alpha \in [0, 2\pi), \cos \alpha = \frac{a}{r}, \sin \alpha = \frac{b}{r}$$

Properties of triangles \rightarrow

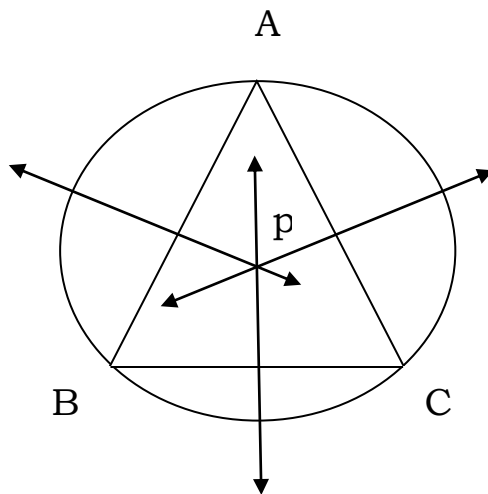


In Center : I

In radius : $ID = IE = IF$

It is INCIRCLE OR DISCIBED CIRCLE

Circumcircle:-



Circumcenter is P,

Circumradius: $PA=PB=PC=R$

SINE RULE:→

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

COSINE RULE :→

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

PROJECTION FORMULA :→

$$a = b \cos C + c \cos B, \quad b = a \cos C + c \cos A, \quad c = a \cos B + b \cos A$$

NAPIER'S FORMULA (TANGENT FORMULA) :→

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot\frac{A}{2}$$

$$\tan\left(\frac{C-A}{2}\right) = \frac{c-b}{c+b} \cot\frac{B}{2}$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot\frac{C}{2}$$

FORMULAE OF SEMI-ANGLES (HALF-ANGLES FORMLE) :→

$$A, B, C \text{ are angles of a triangle} \Rightarrow 0 < A, B, C < \pi \Rightarrow 0 < \frac{A}{2}, \frac{B}{2}, \frac{C}{2} < \frac{\pi}{2}$$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}},$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}},$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{2}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}},$$

$$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}},$$

$$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\tan \frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}},$$

$$\tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

THE AREA OF A TRIANGLE: →

$$\Delta = \frac{1}{2}bc \sin A, \quad \Delta = \frac{1}{2}ac \sin B, \quad \Delta = \frac{1}{2}ab \sin C$$

$$\Delta = \frac{abc}{4R}, \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

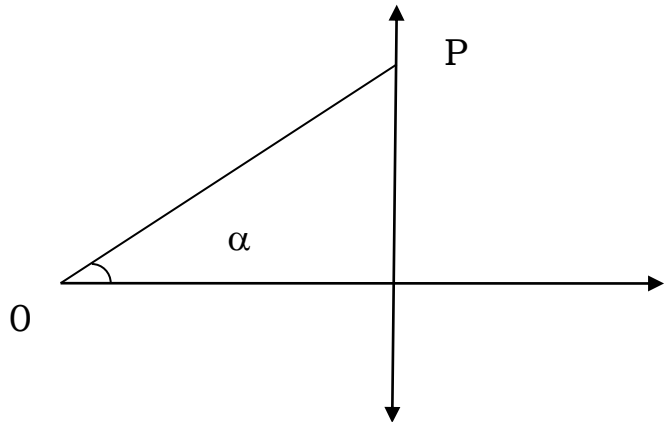
$$\cot A = \frac{b^2 + c^2 - a^2}{4\Delta}, \quad \cot B = \frac{a^2 + c^2 - b^2}{4\Delta}, \quad \cot C = \frac{a^2 + b^2 - c^2}{4\Delta}$$

IN RADIUS : →

$$r = \frac{\Delta}{s}, \quad r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

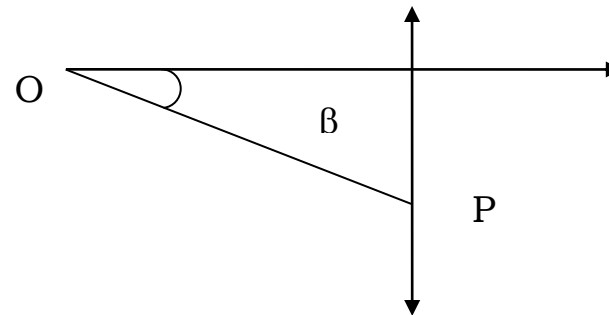
$$r = (s-a) \tan \frac{A}{2}, \quad r = (s-b) \tan \frac{B}{2}, \quad r = (s-c) \tan \frac{C}{2}$$

ANGLE OF ELEVATION :→



α is the angle of elevation of P from O.

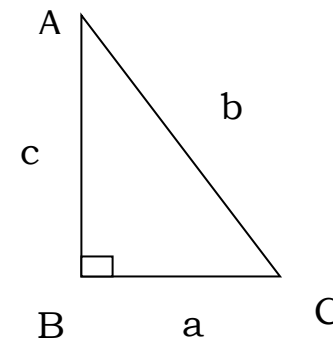
ANGLE OF DEPRESSION:→



β is the angle of depression of P from O.

In $\triangle ABC$, $m\angle B=90^\circ$ using Pythagoras : $b = \sqrt{c^2 + a^2}$

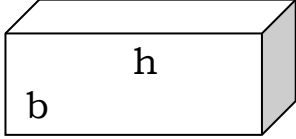
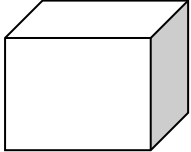
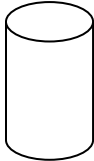
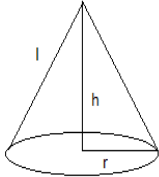
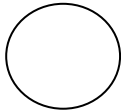
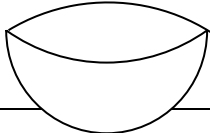
\therefore By sine - rule : $\frac{a}{\sin A} = \frac{b}{1} = \frac{c}{\sin C}$



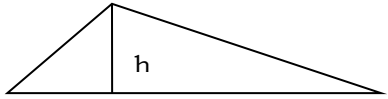
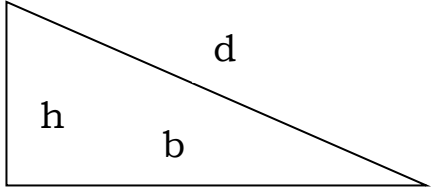
SEQUENCE AND SERIES : →

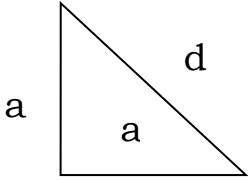
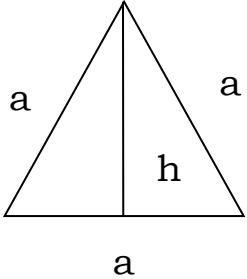

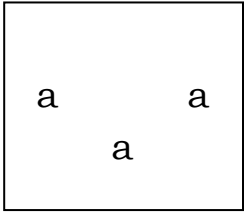
| | SEQUENCE | SERIES | t_n or a_n term | mean between a and b | nth mean between a and b |
|------------------------|---|---|------------------------------|-----------------------|--|
| General | a_1, a_2, \dots, a_n | $S_1 = a_1$ $S_2 = a_1 + a_2$ $S_3 = a_1 + a_2 + a_3$... $S_n = a_1 + a_2 + a_3 + \dots + a_n$ | $a_n = S_n - S_{n-1}$ | ---- | ---- |
| Arithmetic Progression | $a, a+d, a+2d, \dots,$ $a+(n-1)d$ $a = \text{First term}$ $l = \text{last term}$ $= a + (n-1)d$ | $S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$ $= \frac{n}{2} [2a + (n-1)d]$ $= \frac{n}{2} [a + l]$ | $t_n = a + (n-1)d$ | $A = \frac{a+b}{2}$ | $a, A_1, A_2, \dots, A_n, b$ $d = \frac{b-a}{n+1}$ |
| Geometric Progression | $a, ar, ar^2, \dots, ar^{n-1}$ $a = \text{First term}$ | $S_n = a + ar + ar^2 + \dots + ar^{n-1}$ $= \frac{a(r^n - 1)}{r - 1}, r \neq 1$ if $r=1$ then $S_n = na$ | $t_n = ar^{n-1}$ | $G = \sqrt{ab}$ | $a, G_1, G_2, \dots, G_n, b$ $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ |
| | SEQUENCE | SERIES | t_n or a_n term | mean between a and b | nth mean between a and b |
| Harmonic Progression | $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots,$ $\dots, \frac{1}{a+(n-1)d}$ | --- | $t_n = \frac{1}{a + (n-1)d}$ | $H = \frac{2ab}{a+b}$ | $\frac{1}{a}, H_1, H_2, \dots, H_n, \frac{1}{b}$ |

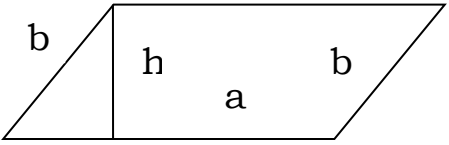
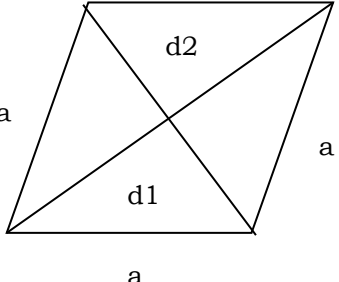
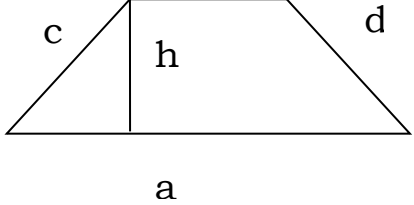
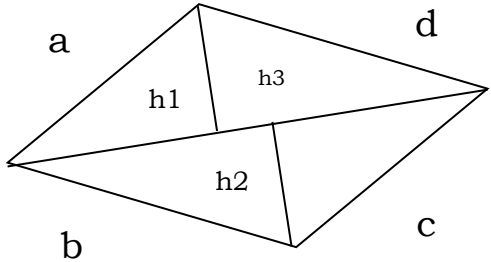
| | | | | | |
|--|--|---|-------------------------------------|------|------|
| Arithmetic and Geometric Progression | $a, (a+d)r, (a+2d)r^2, \dots, (a+(n-1)d)r^{n-1}$ | $S_n = \frac{a}{r-1} + \frac{dr(1-r^n)}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$ | $t_n = \frac{1}{[a+(n-1)d]r^{n-1}}$ | ---- | ---- |
| Relation between A.P., G.P., and H.P. : (1) $AH=G^2$ (2) $A > G > H$ | | | | | |

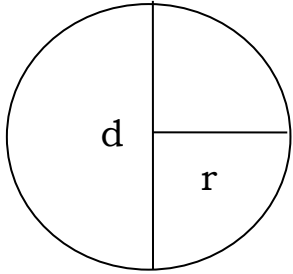
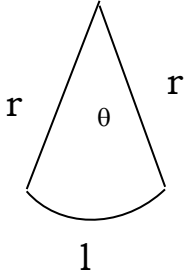
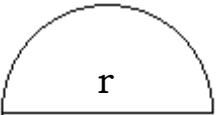
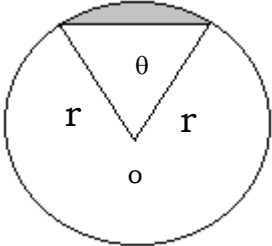
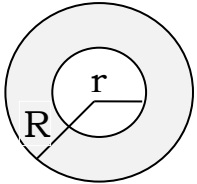
| NAME OF THE SOLID | Figure | LATERAL/ CURVED SRFACE AREA | TOTAL SURFACE AREA | VOLUME | |
|-------------------------------|---|-----------------------------------|--------------------------|-------------------------|--|
| CUBOID |  | $2h(l + b)$ | $2(lb+bh+hl)$ | lbh | |
| CUBE |  | $4a^2$ | $6a^2$ | a^3 | |
| RIGHT CIRCULAR CYLINDER |  | $2\pi rh$ | $2\pi r(r+h)$ | $\pi r^2 h$ | |
| RIGHT CIRCULAR CONE |  | πrl | $\pi r(l+r)$ | $\frac{1}{3} \pi r^2 h$ | |
| SPHERE |  | $4\pi r^2$ | $4\pi r^2$ | $\frac{4}{3} \pi r^3$ | |
| HIMESPHERE |  | $2\pi r^2$ | $3\pi r^2$ | $\frac{2}{3} \pi r^3$ | |

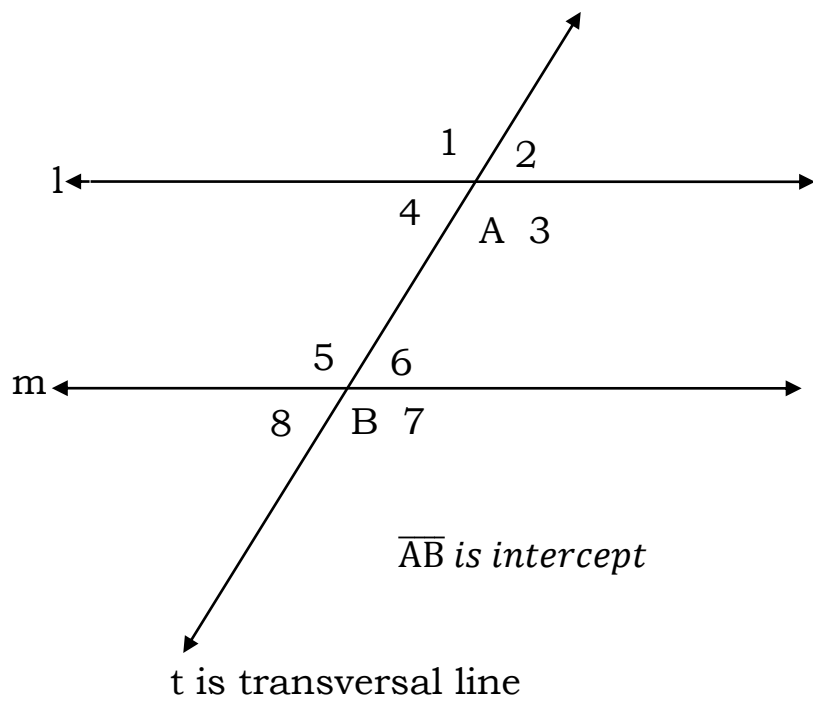
| | | | | | |
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| | | | | | |
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| Sr. No. | Name | Figure | Perimeter in units of length | Area of Square units | Remark |
|---------|----------------|---|------------------------------|--|------------------------|
| 1 | Triangle |  | $a + b + c = 2s$ | $\frac{1}{2}ch$ or $\sqrt{s(s-a)(s-b)(s-c)}$ | |
| 2 | Right Triangle |  | $h+b+d$ | $\frac{1}{2}bh$ | $d = \sqrt{b^2 + h^2}$ |

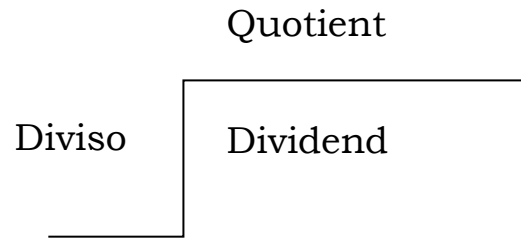
| | | | | | |
|--------|--------------------------------|---|----------|--|--|
| a 3 | Isosceles Right Triangle |  | $2a+d$ | $\frac{1}{2}a^2$ | $d = a\sqrt{2}$ |
| 4 | Equilateral Triangle |  | $3a$ | $\frac{1}{2}ah$ OR $\frac{\sqrt{3}}{4}a^2$ | $h = \frac{\sqrt{3}}{2}a$ |
| 5 | Rectangle |  | $2(l+b)$ | lb | l is length, b is breadth. All angles 90° . Diagonals = & bisect |
| 6 | Square |  | $4a$ | a^2 | a is side. All side = AND $1 \times 90^\circ$ angle. Diagonals = & bisect @ 90° . |

| | | | | | |
|----|---------------|---|------------|-----------------------------|--|
| 7 | Parallelogram |  | $2(a + b)$ | ah | <p>Both opp. angles =.</p> <p>Both opp. sides =.</p> <p>Both opp. Sides \parallel.</p> <p>Diagonals bisect.</p> |
| 8 | Rhombus |  | $4a$ | $\frac{1}{2}d_1d_2$ | <p>d_1d_2 are the diagonal.</p> <p>All sides=.</p> <p>Diagonals bisect @ 90° and \neq.</p> |
| 9 | Trapezium |  | $a+b+c+d$ | $\frac{1}{2}(a + b)h$ | |
| 10 | Quadrilateral |  | $a+b+c+d$ | $\frac{1}{2}(h_1 + h_2)h_3$ | |

| | | | | | |
|----|---|---|--|---|--------------------------------------|
| 11 | Circle |  | $2\pi r$ OR πd | πr^2 | r is radius, d is diameter |
| 12 | Sector of circle |  | $2\pi r \left(\frac{\theta}{360}\right) + 2r$ OR $l + 2r$ | $\pi r^2 \left(\frac{\theta}{360}\right)$ OR $\frac{1}{2}lr$ | l is length of sector |
| 13 | Semi-circle |  | $\pi r + 2r$ | $\frac{1}{2}\pi r^2$ | |
| 14 | Segment of a Circle (Shaded portion) |  | $\left[2\pi r \left(\frac{\theta}{360}\right) + 2r \sin \frac{\theta}{2}\right]$ | $\left[\pi r^2 \left(\frac{\theta}{360}\right) - \frac{1}{2}r^2 \sin \theta\right]$ | |
| 15 | Ring (Shaded region) |  | - | $\pi(R^2 - r^2)$ | R is Outer radius, r is inner radius |



| Alternate Angles | Corresponding Angles | Interior Angles |
|---------------------------|---------------------------|-------------------------------------|
| $\angle 3 \cong \angle 5$ | $\angle 1 \cong \angle 5$ | $m\angle 4 + m\angle 5 = 180^\circ$ |
| $\angle 4 \cong \angle 6$ | $\angle 2 \cong \angle 6$ | $m\angle 3 + m\angle 6 = 180^\circ$ |
| | $\angle 3 \cong \angle 7$ | |
| | $\angle 4 \cong \angle 8$ | |

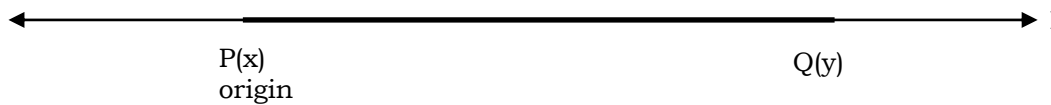


| |
|---|
| <i>Numerator</i> |
| <hr style="width: 80%; margin: 0 auto;"/> |
| <i>Denominator</i> |

Co-ordinate system in \mathbb{R}^1 :

$$\text{Set : } \{ P(x) / x \in \mathbb{R} \}$$

Distance Function in \mathbb{R}^1 : $P(x), Q(y) \in \mathbb{R}^1$, The Function $d: \mathbb{R}^1 \times \mathbb{R}^1 \rightarrow \mathbb{R}^+ \cup \{0\}$, $d(P, Q) = |x - y|$

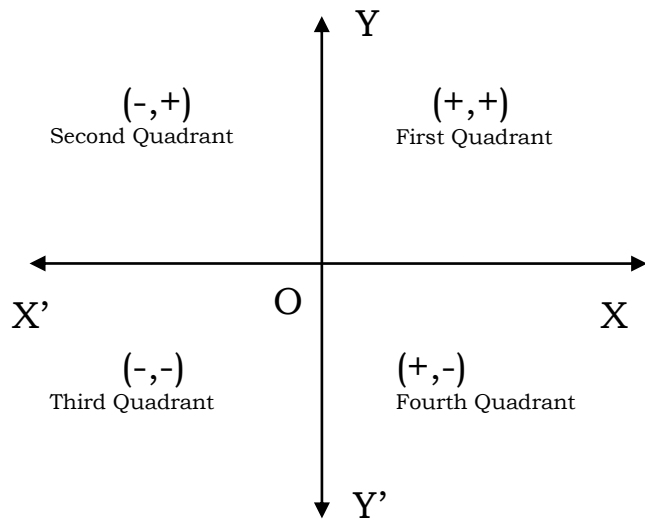


Co-ordinate system in \mathbb{R}^2 :

$$\text{Set : } \{ P(x, y) / x \in \mathbb{R}, y \in \mathbb{R} \}$$

Distance Function in \mathbb{R}^2 :

$P(x_1, y_1), Q(x_2, y_2) \in \mathbb{R}^2$, The Function $d: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}^+ \cup \{0\}$, $d(P, Q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$



The line $\overleftrightarrow{X'X}$ is called X-axis \rightarrow Horizontal line

X-axis = $\{ P (x, 0) / x \in \mathbb{R} \}$

The line $\overleftrightarrow{Y'Y}$ is called Y-axis \rightarrow Vertical line

Y-axis = $\{ P (0, y) / y \in \mathbb{R} \}$

The point O is called the Origin.

The First Quadrant = $\{ P (x, y) / x > 0, y > 0 \}$

The Second Quadrant = $\{ P (x, y) / x < 0, y > 0 \}$

The Third Quadrant = $\{ P (x, y) / x < 0, y < 0 \}$

The Fourth Quadrant = $\{ P (x, y) / x > 0, y < 0 \}$

If P is $P (a, 0)$, $a \in \mathbb{R}^+$ then the +ve direction of X-axis is $\text{dir } \overrightarrow{OP}$.

If Q is $Q (0, b)$, $b \in \mathbb{R}^+$ then the +ve direction of Y-axis is $\text{dir } \overrightarrow{OQ}$.

The two axes are perpendicular to each other, this co-ordinate system is called the Rectangular Co-ordinate System.

Distance Function in R^2 & R^1 Properties:

$\forall P, Q, T \in R^2 \text{ \& } R^1$

(1) $d(P, Q) \geq 0$

(2) $d(P, Q) = 0 \Leftrightarrow P = Q$

(3) $d(P, Q) = d(Q, P)$

(4) $d(P, Q) \leq d(P, T) + d(T, Q)$ (Triangular inequality)

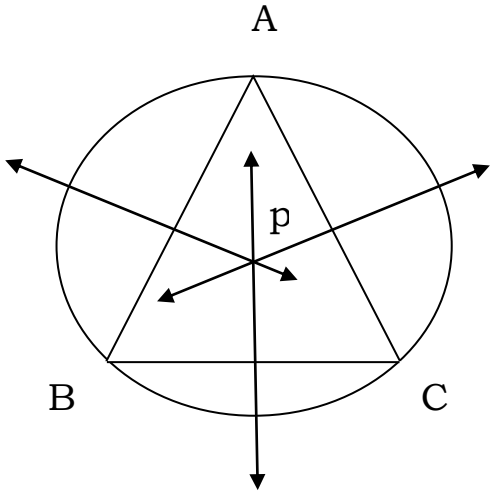
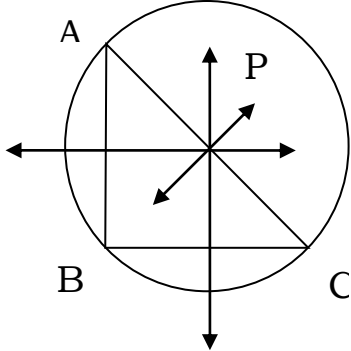
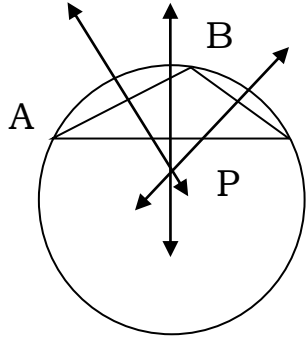
Identification of Geometrical Figures & Properties:

Type of triangles:

| Sr.No. | Side | angle |
|--------|-----------------------------|---|
| 1 | Isosceles : any two sides = | Equiangular: all angles= |
| 2 | Equilateral : all sides = | Acute angled: any one angle $< 90^\circ$ |
| 3 | Scalene: all sides \neq | Right angled: any one angle = 90° |
| 4 | | Obtuse angled: any one angle $> 90^\circ$ |

Circumcentre(P): The perpendicular bisector of three side of a triangle is concurrent. The point of concurrency is called circumcentre of a triangle.

Circumcircle(P) :- In ΔABC , co-ordinate of $A(x_1, y_1)$, co-ordinate of $B(x_2, y_2)$, co-ordinate of $C(x_3, y_3)$ and co-ordinate of $P(x, y)$, $PA = PB = PC = \text{radius of circle}$, P is circumcentre.

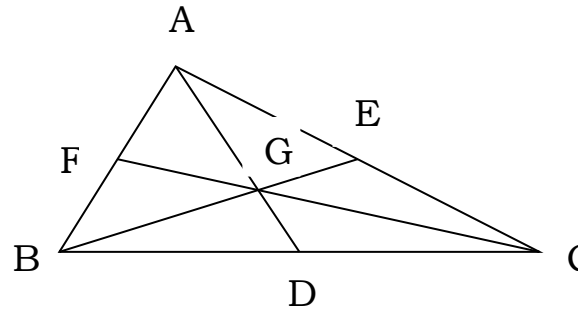
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|  |  |  |
| <p>The Circumcentre(P) of an acute angled triangle is inside the triangle.</p> | <p>The Circumcentre(P) of a right angled triangle is mid-point of the hypotenuse.</p> | <p>The Circumcentre(P) of an obtuse angled triangle is outside the triangle.</p> |

Centroid of a triangle (G): -

The point of concurrence of the three medians of a triangle is called centroid.

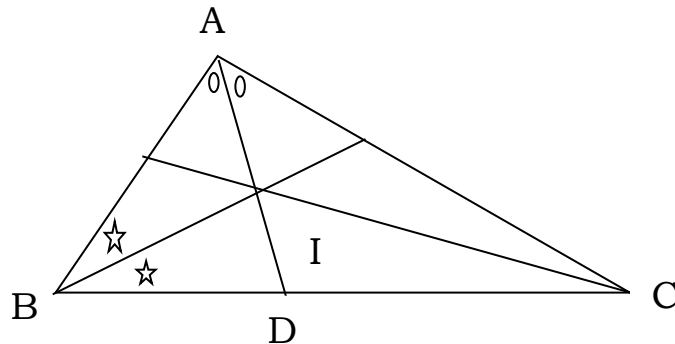
$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$\overline{AD}, \overline{BE}, \overline{CF}$ are median of triangle ABC



The Incentre of triangle (I) : -

The point of concurrence of the bisectors of the three angles of triangle is called incentre of triangle.



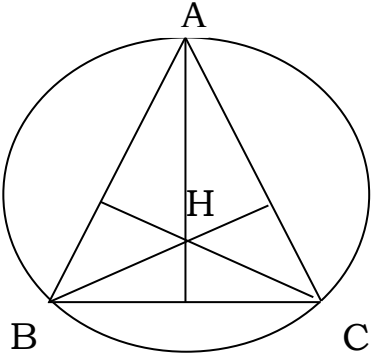
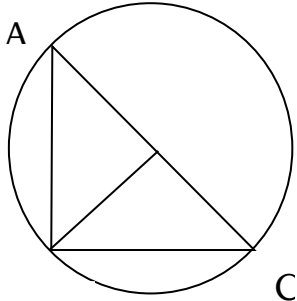
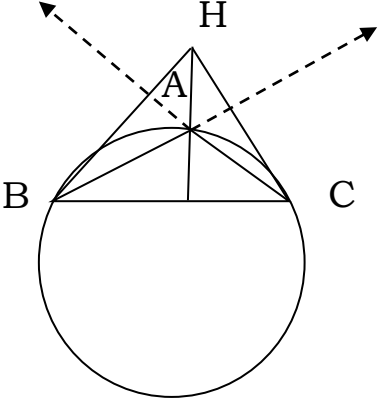
$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

$$AB = c, BC = a, AC = b,$$

$$\frac{AB}{AC} = \frac{BD}{DC} = \frac{c}{b} = \lambda$$

Orthocentre of triangle (H) :-

The point of concurrence of the altitudes of a triangle is called its Orthocenter.

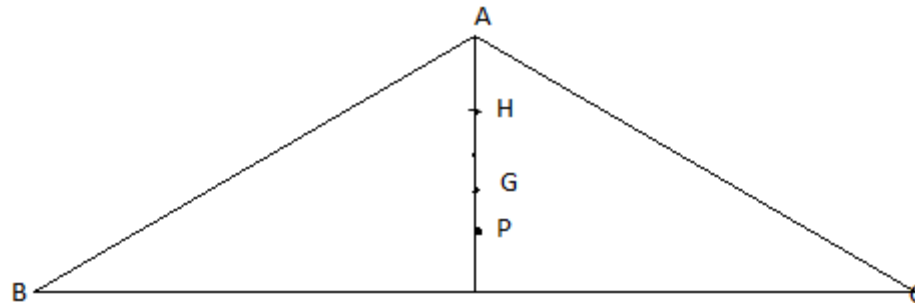
| | | |
|--|---|--|
|  |  |  |
| <p>If triangle is an acute angled triangle then H in the interior of triangle.</p> | <p>If triangle is right angled triangle then the H is the vertex forming the right angle of the triangle.</p> | <p>If triangle is an obtuse angled triangle then H is in the exterior of triangle.</p> |

Short Cut Method : -

1. If (x_1, y_1) , (x_2, y_2) are the ends of the hypotenuse of a right angled isosceles triangle, then the third vertex is given by $\left(\frac{x_1+x_2 \pm (y_1-y_2)}{2}, \frac{y_1+y_2 \mp (x_1-x_2)}{2}\right)$.
2. Given two vertices (x_1, y_1) and (x_2, y_2) of an equilateral triangle, then its third vertex is given by $\left(\frac{x_1+x_2 \pm \sqrt{3}(y_1-y_2)}{2}, \frac{y_1+y_2 \mp \sqrt{3}(x_1-x_2)}{2}\right)$.
3. Area of the triangle formed by (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{vmatrix}.$$
4. Area of the quadrilateral formed by (x_1, y_1) , (x_2, y_2) , (x_3, y_3) and (x_4, y_4) is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}.$$
5. If ABCD is a parallelogram, then $D = A - B + C$.
6. If D, E, F are the mid-point of the sides BC, CA, AB of ΔABC , then $A = E + F - D$, $B = F + D - E$, $C = D + E - F$.
7. If D, E, F are the mid-point of the sides BC, CA, AB of ΔABC , then The Centroid of $\Delta ABC =$ Centroid of ΔDEF .
8. Orthocentre, Centroid, Circumcentre of triangle are collinear, Centroid divides the line joining Orthocentre and Circumcentre in the ratio 2:1.
9. In An Equilateral triangle Orthocentre, Centroid, Circumcentre, incentre coincide.
10. A triangle is isosceles if any two of its medians are equal.
11. In a triangle has integral co-ordinate then it will never be Equilateral.
12. In an Isosceles triangle Orthocentre, Centroid, Circumcentre lies on the same line.(median)



13. In Right Angled triangle if sum of the square of two sides is equal to the square of the square of hypotenuse.
14. In right angled triangle the middle point of the hypotenuse is circumcentre of triangle.
15. In an obtuse angled triangle orthocenter and circumcentre are always outside the triangle.